Letter

Recursive Fault Estimation With Energy Harvesting Sensors and Uniform Quantization Effects

Yu-Ang Wang, Bo Shen, *Senior Member, IEEE*, and Lei Zou, *Senior Member, IEEE*

Dear Editor,

In this letter, the recursive fault estimation issue is considered for nonlinear time-varying systems subject to the effects induced by energy harvesting sensors and uniform quantization. Based on the energy harvesting mechanism and stochastic distribution of the absorbed energy, the real-time occurrence probability of missing measurements is calculated recursively. This research intends to develop a recursive estimator for the considered nonlinear time-varying system with energy harvesting sensors, such that, under uniform quantization effects, the state and fault can be jointly estimated. By adopting the induction approach, an upper bound is firstly calculated for the estimation error covariances (EECs) of the state and fault. Then, the value of the time-varying estimator parameter is computed through minimizing such calculated upper bound. In the end, an illustrative example is presented to verify the availability of the developed fault estimation method.

Fault estimation, which is a significant research issue in fault diagnosis field, has gained an ever-increasing research attention in recent decades. The main purpose of fault estimation is to estimate the "shape" and "size" for the underlying fault signal according to the available information (e.g., system model, received measurements, priori knowledge about the faults). So far, many interesting results concerning the fault estimation problems of different systems have been reported in the literature, see e.g., [1]–[3].

Quantization is considered to be an important source of the network systems performance. Such a research topic has attracted considerable research interest in recent years, see e.g., [4], [5]. To date, two kinds of quantization methods (i.e., the uniform-type quantization and the logarithmic-type quantization) have been adopted in the past literatures. In engineering practice, more and more energy harvesting sensors (EHS) are applied in practical systems for the aim of reducing the restriction of limited battery capacity on communication networks and providing permanent energy supply for remote devices. Under the effects of energy harvesting, a set of "rechargeable batteries" are adopted in sensors to store the energy absorbing from external environment (e.g., solar panels and wind mills) [6]. Nevertheless, the utilization of energy harvesting technique would give rise to certain distinguished phenomenon. More specifically, the measurement of sensor would be discarded if there is no energy stored in the sensor. The energy harvesting does result in measurement losses which, if not adequately tackled, would largely affect the filtering/control performance. So far, some preliminary results concerning the state estimation (or filtering) problem subject to EHS have been reported, see e.g., [6], [7].

Corresponding author: Bo Shen.

Citation: Y.-A. Wang, B. Shen, and L. Zou, "Recursive fault estimation with energy harvesting sensors and uniform quantization effects," *IEEE/CAA J. Autom. Sinica*, vol. 9, no. 5, pp. 926–929, May 2022.

The authors are with the College of Information Science and Technology, Donghua University, Shanghai 201620, and also with the Engineering Research Center of Digitalized Textile and Fashion Technology, Ministry of Education, Shanghai 201620, China (e-mail: yuangwang98@163.com; bo.shen@dhu.edu.cn; zouleicup@gmail.com).

Color versions of one or more of the figures in this paper are available online at http://ieeexplore.ieee.org.

Digital Object Identifier 10.1109/JAS.2022.105572

Compared with the existing results on fault estimation problem, in this letter, we shall thoroughly consider the impacts induced by the energy harvesting mechanism, and discuss the estimation design and estimation performance analysis issues according to such impacts. Two major challenges in this letter are identified as follows: 1) How to develop a fault estimation scheme to handle nonlinearities and uniform quantization effects (UQE) under the EHS constraints? and 2) How to examine the transient behavior of the states and fault estimation error for the time-varying nature of the energy stored in the sensor? The main contributions of this work are highlighted as follows: 1) The fault estimation issue is, for the first time, studied for time-varying system (TVS) with EHS and UQE; 2) An upper bound (UB) of the estimation error covariance (EEC) is calculated recursively based on two coupled Riccati-like difference equations; and 3) The estimator gain matrix is calculated through minimizing the trace of the resultant EEC.

Problem formulation:

System model and communication network:

The considered TVS is given with the following form:

$$\begin{cases} x_{s+1} = l(x_s) + \mathcal{A}_s x_s + \mathcal{B}_s \omega_s + \mathcal{F}_s f_s \\ y_s = \mathcal{G}_s x_s + \mathcal{D}_s \nu_s \end{cases} \tag{1}$$

where $y_s \in \mathbb{R}^{n_y}$ and $x_s \in \mathbb{R}^{n_x}$ represent the measurement output vector and the system state, respectively; $\omega_s \in \mathbb{R}^{n_\omega}$ stands for the process noise and $\nu_s \in \mathbb{R}^{n_\nu}$ denotes the measurement noise; $l(\cdot) : \mathbb{R}^{n_x} \to \mathbb{R}^{n_x}$ is a nonlinear function which will be introduced later. $\mathcal{A}_s$, $\mathcal{B}_s$, $\mathcal{G}_s$, $\mathcal{D}_s$ and $\mathcal{F}_s$ are time-varying matrices.

To further characterize the considered TVS, we introduce the following assumptions.

Assumption 1 [8]: Considering the nonlinearity $l(\cdot)$, there exists a positive constant $\Im$ such that the conditions $l(0) = 0$ and $\|l(\jmath) - l(\imath)\| \le \Im \|\jmath - \imath\|$ holds for all $\jmath, \imath \in \mathbb{R}^{n_\imath}$.

Assumption 2: The initial value x_0, the noises ω_s and ν_s are mutually independent with the following statistical properties:

$$E\{\omega_j\} = E\{\nu_j\} = 0, \ \ E\{x_0\} = \bar{x}_0, \ \ E\{x_0 x_0^T\} = P_{0|0}$$

$$E\{\omega_i \omega_j^T\} = \delta(i-j) W_i, \ \ E\{\nu_i \nu_j^T\} = \delta(i-j) V_i \tag{2}$$

where $W_i > 0$, $V_i > 0$ and $P_{0|0} > 0$ are known matrices, which have the appropriate dimensions. In practical applications, the values of W_i, V_i and $P_{0|0}$ are obtained according to the preliminary knowledge of the process noise, measurement noise and the initial state. Sometimes it is difficult to obtain the exact values of these covariance matrices. In this situation, some relatively conservative matrices can be utilized to give the upper-bounds for the unknown covariance matrices.

Assumption 3 [9]: The fault signal $f_s \in \mathbb{R}^{n_f}$ satisfies $\Delta(\Delta(f_s)) = 0$, $E\{f_0\} = E\{\Delta(f_0)\} = 0$ with $\Delta(f_s) \triangleq f_{s+1} - f_s$.

Taking the UQE into account, the uniform quantization process is written as

$$\vec{y}_s = \mathfrak{Y}(y_s) \triangleq \left[(\ell \Re \frac{y_{1,s}}{\ell})^T \ \ \cdots \ \ (\ell \Re \frac{y_{n_y,s}}{\ell})^T \right]^T \tag{3}$$

where $y_{i,s}$ $(i = 1, 2, \ldots, n_y)$ is the i-th entry in y_s and ℓ is referred as the quantization level. The function $\Re(\cdot)$ rounds a real-number to the nearest integer number. The uniform quantization error is defined as $\sigma_s \triangleq \mathfrak{Y}(y_s) - y_s$. It is easy to see that $\|\sigma_s\|_\infty \le \ell/2$.

Energy harvesting model:

Let $z_s^r \in \{S_r, S_{r-1}, ..., 0\}$ and $h_s^r \in \mathbb{N}^+$ be the energy level stored in sensor $r \in \{1, 2, \ldots, n_y\}$ and the energy units that sensor r can harvest at time instant s, respectively, in which S_r indicates the maximum limit for the energy storage of sensor r. For any $i \in \{1, 2, \ldots, n_y\}$, $\{h_s^i\}_{s \ge 0}$ is assumed to be a sequence of independent identically distributed random variables whose probability distribution is given by

$$\text{Prob}\{h_s^i = j\} = p_j, \quad j = 0,1,2\ldots \tag{4}$$

where p_j satisfies $0 \le p_j \le 1$ and $\sum_{j=0}^{+\infty} p_j = 1$.

Define the indicator variable Υ_s as

$$\Upsilon_s \triangleq \text{diag}\{\Upsilon_{1,\{z_s^1>0\}}, \Upsilon_{2,\{z_s^2>0\}}, \ldots, \Upsilon_{n_y,\{z_s^{n_y}>0\}}\} \tag{5}$$

where $\Upsilon_{i,\{z_s^i>0\}} \triangleq \begin{cases} 1, & z_s^i > 0 \\ 0, & \text{otherwise.} \end{cases}$

Under the energy harvesting mechanism, a sensor would harvest the energy from external environment and store it in the battery. When the stored unit energy is non-zero, the sensor would transmit its current measurement signal to the fault estimator and consume one unit energy. Accordingly, the dynamics of z_s^i can be written as

$$\begin{cases} z_{s+1}^i = \max\{\min\{z_s^i + h_s^i - \Upsilon_{i,\{z_s^i>0\}}, S_i\}, 0\} \\ z_0^i \le S_i \end{cases} \tag{6}$$

and $\tilde{y}_s = \Upsilon_s \vec{y}_s$ is the measurement received by the remote fault estimator.

Fault estimator:

Define $\bar{x}_s \triangleq \begin{bmatrix} x_s^T & f_s^T & \Delta^T(f_s) \end{bmatrix}^T$. Based on (1), we formulate an augmented system of the following form:

$$\begin{cases} \bar{x}_{s+1} = \bar{\mathcal{A}}_s \bar{x}_s + l(\bar{x}_s) + \bar{\mathcal{B}}_s \omega_s \\ y_s = \bar{\mathcal{G}}_s \bar{x}_s + \bar{\mathcal{D}}_s \nu_s \end{cases} \tag{7}$$

where

$$\bar{\mathcal{A}}_s \triangleq \begin{bmatrix} \mathcal{A}_s & \mathcal{F}_s & 0 \\ 0 & I & I \\ 0 & 0 & I \end{bmatrix}, \quad \bar{\mathcal{B}}_s \triangleq \begin{bmatrix} \mathcal{B}_s \\ 0 \\ 0 \end{bmatrix}, \quad l(\bar{x}_s) \triangleq \begin{bmatrix} l(x_s) \\ 0 \\ 0 \end{bmatrix}$$

$$\bar{\mathcal{G}}_s \triangleq [\mathcal{G}_s \quad 0 \quad 0], \quad \bar{\mathcal{D}}_s \triangleq \mathcal{D}_s.$$

The fault estimator for the augmented system (7) with the available measurement is constructed as follows:

$$\begin{cases} \hat{x}_{s+1|s} = \bar{\mathcal{A}}_s \hat{x}_{s|s} + l(\hat{x}_{s|s}) \\ \hat{x}_{s+1|s+1} = \hat{x}_{s+1|s} + \mathcal{K}_{s+1}(\tilde{y}_{s+1} - \Lambda_{s+1}\bar{\mathcal{G}}_{s+1}\hat{x}_{s+1|s}) \\ \hat{x}_{0|0} = \begin{bmatrix} \bar{x}_0^T & 0 & 0 \end{bmatrix}^T \end{cases} \tag{8}$$

where $\hat{x}_{s+1|s}$ and $\hat{x}_{s|s}$ represent the one-step prediction (OSP) and the estimate of x_s at time s, respectively; $\mathcal{K}_{s+1}$ is the time-varying estimator gain to be designed; and $\Lambda_s \triangleq \mathbb{E}\{\Upsilon_s\}$.

Letting $e_{s+1|s} \triangleq \bar{x}_{s+1} - \hat{x}_{s+1|s}$ be the OSP error and $e_{s+1|s+1} \triangleq \bar{x}_{s+1} - \hat{x}_{s+1|s+1}$ be the estimation error, we have

$$\begin{cases} e_{s+1|s} = \bar{\mathcal{A}}_s e_{s|s} + l(e_{s|s}) + \bar{\mathcal{B}}_s \omega_s \\ e_{s+1|s+1} = e_{s+1|s} - \mathcal{K}_{s+1}(\tilde{y}_{s+1} - \Lambda_{s+1}\bar{\mathcal{G}}_{s+1}\hat{x}_{s+1|s}) \end{cases} \tag{9}$$

where $l(e_{s|s}) \triangleq l(\bar{x}_s) - l(\hat{x}_{s|s})$.

Our objective in this letter is to construct a fault estimator according to (8) such that:

1) An UB $\bar{\aleph}_{s|s}$ of the EEC $\Gamma_{s|s} \triangleq \mathbb{E}\{e_{s|s}e_{s|s}^T\}$ can be guaranteed;

2) An appropriate gain of fault estimator $\mathcal{K}_{s+1}$ is designed recursively to minimize the trace of the UB $\bar{\aleph}_{s|s}$.

Main results: In this section, we shall calculate the UB of the EEC firstly. Then, we are going to compute the value of the fault estimator gain matrix through minimizing such an UB.

The following lemmas are necessary for the derivation of our main results.

Lemma 1 [10]: Consider the energy level $\{z_s^i\}_{s\ge 0}$ with the probability distribution given by (6). Then, by letting $\theta_s^i \triangleq [\text{Prob}(z_s^i = 0) \;\; \text{Prob}(z_s^i = 1) \;\; \cdots \;\; \text{Prob}(z_s^i = S_i)]^T$, we have

$$\begin{cases} \theta_{s+1}^i = \delta_i + \Omega_i \theta_s^i \\ \theta_0^i = [\underbrace{0 \;\; \cdots \;\; 0}_{z_0^i} \;\; 1 \;\; \underbrace{0 \;\; \cdots \;\; 0}_{S_i - z_0^i}]^T \end{cases} \tag{10}$$

where $\delta_i \triangleq [\underbrace{0 \;\; \cdots \;\; 0}_{S_i} \;\; 1]^T$ and

$$\Omega_i = -\begin{bmatrix} -p_0 & -p_0 & 0 & \cdots & 0 \\ -p_1 & -p_1 & -p_0 & \cdots & 0 \\ -p_2 & -p_2 & -p_1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ -p_{S_i-1} & -p_{S_i-1} & -p_{S_i-2} & \cdots & -p_0 \\ \sum_{j=0}^{S_i-1} p_j & \sum_{j=0}^{S_i-1} p_j & \sum_{j=0}^{S_i-2} p_j & \cdots & p_0 \end{bmatrix}. \tag{11}$$

From Lemma 1, it is easy to see that the transmission probability of the measurement at time instant s can be calculated by $\Lambda_s \triangleq \text{diag}\{\lambda_{1,s}, \lambda_{2,s}, \ldots, \lambda_{n_y,s}\}$ in which $\lambda_{i,s} \triangleq \text{Prob}(\Upsilon_{i,\{z_s^i\}} = 1) = [0 \;\; \underbrace{1 \;\; \cdots \;\; 1}_{S_i}]\theta_s^i$.

Next, according to the estimation error dynamics (9), we are going to derive the covariances of the OSP error and estimation error. Then, an UB of the EEC would be given.

Theorem 1: Given four positive scalars $\beta_1, \beta_2, \beta_3, \beta_4, \beta_5$ and γ, let us calculate the matrices $\{\bar{\aleph}_{s+1|s}\}_{s\ge 0}$ and $\{\bar{\aleph}_{s+1|s+1}\}_{s\ge 0}$ recursively as follows:

$$\bar{\aleph}_{s+1|s} = (1+\beta_1)\bar{\mathcal{A}}_s \bar{\aleph}_{s|s} \bar{\mathcal{A}}_s^T + \bar{\mathcal{B}}_s W_s \bar{\mathcal{B}}_s^T + (1+\beta_1^{-1})\Im^2 \text{tr}\{\bar{\aleph}_{s|s}\} I \tag{12}$$

$$\begin{aligned} \bar{\aleph}_{s+1|s+1} = & (1+\beta_2+\beta_3)(I - \mathcal{K}_{s+1}\Lambda_{s+1}\bar{\mathcal{G}}_{s+1})\bar{\aleph}_{s+1|s} \\ & \times (I - \mathcal{K}_{s+1}\Lambda_{s+1}\bar{\mathcal{G}}_{s+1})^T + (1+\beta_2^{-1}+\beta_4+\beta_5) \\ & \times \mathcal{K}_{s+1}(\Lambda_{s+1} \circ (n_y \ell^2/4) I)\mathcal{K}_{s+1}^T + (1+\gamma) \\ & \times (1+\beta_3^{-1}+\beta_4^{-1})\mathcal{K}_{s+1}((\Lambda_{s+1} - \Lambda_{s+1}^2) \circ \bar{\mathcal{G}}_{s+1} \\ & \times \bar{\aleph}_{s+1|s}\bar{\mathcal{G}}_{s+1}^T)\mathcal{K}_{s+1}^T + (1+\beta_3^{-1}+\beta_4^{-1}) \\ & \times (1+\gamma^{-1})\mathcal{K}_{s+1}((\Lambda_{s+1} - \Lambda_{s+1}^2) \circ \bar{\mathcal{G}}_{s+1} \\ & \times \hat{x}_{s+1|s}\hat{x}_{s+1|s}^T \bar{\mathcal{G}}_{s+1}^T)\mathcal{K}_{s+1}^T + (1+\beta_5^{-1}) \\ & \times \mathcal{K}_{s+1}(\Lambda_{s+1} \circ \bar{\mathcal{D}}_{s+1} V_{s+1} \bar{\mathcal{D}}_{s+1}^T)\mathcal{K}_{s+1}^T \end{aligned} \tag{13}$$

with the initial matrix $\bar{\aleph}_{0|0} \triangleq \text{diag}\{P_{0|0}, 0, 0\}$, where $\circ$ represents the Hadamard product. Then, $\bar{\aleph}_{s+1|s+1}$ is an UB of the EEC $\Gamma_{s+1|s+1} \triangleq E\{e_{s+1|s+1}e_{s+1|s+1}^T\}$.

Proof: From (9), the OSP error covariance $\Gamma_{s+1|s} \triangleq E\{e_{s+1|s}e_{s+1|s}^T\}$ can be computed as follows:

$$\begin{aligned} \Gamma_{s+1|s} = & E\{e_{s+1|s}e_{s+1|s}^T\} \\ = & \bar{\mathcal{A}}_s \Gamma_{s|s} \bar{\mathcal{A}}_s^T + E\{\bar{\mathcal{A}}_s e_{s|s} l^T(e_{s|s}) + \bar{\mathcal{B}}_s W_s \bar{\mathcal{B}}_s^T \\ & + l(e_{s|s}) e_{s|s}^T \bar{\mathcal{A}}_s^T\} + E\{l(e_{s|s}) l^T(e_{s|s})\}. \end{aligned} \tag{14}$$

Based on Lemma 3 in [8], we have

$$\begin{aligned} E\{\bar{\mathcal{A}}_s e_{s|s} l^T(e_{s|s}) + l(e_{s|s}) e_{s|s}^T \bar{\mathcal{A}}_s^T\} \\ \le \beta_1 \bar{\mathcal{A}}_s \Gamma_{s|s} \bar{\mathcal{A}}_s^T + \beta_1^{-1} E\{l(e_{s|s}) l^T(e_{s|s})\}. \end{aligned} \tag{15}$$

According to Assumption 1, the term $E\{l(e_{s|s}) l(e_{s|s})^T\}$ can be derived as follows:

$$
\begin{aligned}
E\{l(e_{s|s})l(e_{s|s})^T\} &\le E\{l^T(e_{s|s})l(e_{s|s})I\}\\
&\le \Im^2 E\{e_{s|s}^T e_{s|s} I\} \le \Im^2 E\{\mathrm{tr}(\Gamma_{s|s})\}I. \qquad (16)
\end{aligned}
$$

Then, with (15) and (16), we have

$$
\begin{aligned}
\Gamma_{s+1|s} \le\ & (1+\beta_1)\bar{\mathcal{A}}_s\Gamma_{s|s}\bar{\mathcal{A}}_s^T + \bar{\mathcal{B}}_s W_s \bar{\mathcal{B}}_s^T\\
&+ (1+\beta_1^{-1})\Im^2 \mathrm{tr}\{\Gamma_{s|s}\}I. \qquad (17)
\end{aligned}
$$

which implies $\Gamma_{s+1|s} \le \bar{\aleph}_{s+1|s}$.

Similarly, by using Lemmas 1 and 3 in [11], the EEC $\Gamma_{s+1|s+1}$ can be calculated as follows:

$$
\begin{aligned}
\Gamma_{s+1|s+1} =\ & (I-\mathcal{K}_{s+1}\Lambda_{s+1}\bar{\mathcal{G}}_{s+1})\Gamma_{s+1|s}(I-\mathcal{K}_{s+1}\Lambda_{s+1}\bar{\mathcal{G}}_{s+1})^T\\
&+\mathcal{K}_{s+1}(\Lambda_{s+1}\circ E\{\sigma_{s+1}\sigma_{s+1}^T\})\mathcal{K}_{s+1}^T + \mathcal{K}_{s+1}\\
&\times((\Lambda_{s+1}-\Lambda_{s+1}^2)\circ\bar{\mathcal{G}}_{s+1}E\{\bar{x}_{s+1}\bar{x}_{s+1}^T\}\bar{\mathcal{G}}_{s+1}^T)\mathcal{K}_{s+1}^T\\
&+\mathcal{K}_{s+1}(\Lambda_{s+1}\circ\bar{\mathcal{D}}_{s+1}V_{s+1}\bar{\mathcal{D}}_{s+1}^T)\mathcal{K}_{s+1}^T\\
&-L_{1,s+1}-L_{1,s+1}^T-L_{2,s+1}-L_{2,s+1}^T\\
&+L_{3,s+1}+L_{3,s+1}^T+L_{4,s+1}+L_{4,s+1}^T \qquad (18)
\end{aligned}
$$

where

$$
\begin{aligned}
L_{1,s+1} &= E\{(I-\mathcal{K}_{s+1}\Lambda_{s+1}\bar{\mathcal{G}}_{s+1})e_{s+1|s}\\
&\qquad\times\sigma_{s+1}^T\Upsilon_{s+1}^T\mathcal{K}_{s+1}^T\}\\
L_{2,s+1} &= E\{(I-\mathcal{K}_{s+1}\Lambda_{s+1}\bar{\mathcal{G}}_{s+1})e_{s+1|s}\\
&\qquad\times\bar{x}_{s+1}^T\bar{\mathcal{G}}_{s+1}^T(\Upsilon_{s+1}-\Lambda_{s+1})^T\mathcal{K}_{s+1}^T\}\\
L_{3,s+1} &= E\{\mathcal{K}_{s+1}\Upsilon_{s+1}\sigma_{s+1}\\
&\qquad\times\bar{x}_{s+1}^T\bar{\mathcal{G}}_{s+1}^T(\Upsilon_{s+1}-\Lambda_{s+1})^T\mathcal{K}_{s+1}^T\}\\
L_{4,s+1} &= E\{\mathcal{K}_{s+1}\Upsilon_s\sigma_{s+1}\nu_{s+1}^T\bar{\mathcal{D}}_{s+1}^T\Upsilon_{s+1}^T\mathcal{K}_{s+1}^T\}.
\end{aligned}
$$

Considering the constraint on the uniform quantization error, we have

$$
E\{\sigma_{s+1}\sigma_{s+1}^T\} \le E\{\sigma_{s+1}^T\sigma_{s+1}I\} \le 0.25 n_y \ell^2 I. \qquad (19)
$$

By using Lemma 3 in [8], we can rewrite $\mathbb{E}\{\bar{x}_{s+1}\bar{x}_{s+1}^T\}$ as follows:

$$
\begin{aligned}
E\{\bar{x}_{s+1}\bar{x}_{s+1}^T\} &= E\{(e_{s+1|s}+\hat{x}_{s+1|s})(e_{s+1|s}+\hat{x}_{s+1|s})^T\}\\
&\le (1+\gamma)\Gamma_{s+1|s}+(1+\gamma^{-1})\hat{x}_{s+1|s}\hat{x}_{s+1|s}^T. \qquad (20)
\end{aligned}
$$

By using Lemma 3 in [8] and Lemma 3 in [11] again and combining (18)−(20), we have

$$
\begin{aligned}
\Gamma_{s+1|s+1} \le\ & (1+\beta_2+\beta_3)(I-\mathcal{K}_{s+1}\Lambda_{s+1}\bar{\mathcal{G}}_{s+1})\bar{\aleph}_{s+1|s}\\
&\times(I-\mathcal{K}_{s+1}\Lambda_{s+1}\bar{\mathcal{G}}_{s+1})^T + (1+\beta_2^{-1}+\beta_4+\beta_5)\\
&\times\mathcal{K}_{s+1}(\Lambda_{s+1}\circ(n_y\ell^2/4)I)\mathcal{K}_{s+1}^T + (1+\gamma)\\
&\times(1+\beta_3^{-1}+\beta_4^{-1})\mathcal{K}_{s+1}((\Lambda_{s+1}-\Lambda_{s+1}^2)\circ\bar{\mathcal{G}}_{s+1}\\
&\times\bar{\aleph}_{s+1|s}\bar{\mathcal{G}}_{s+1}^T)\mathcal{K}_{s+1}^T + (1+\beta_3^{-1}+\beta_4^{-1})\\
&\times(1+\gamma^{-1})\mathcal{K}_{s+1}((\Lambda_{s+1}-\Lambda_{s+1}^2)\circ\bar{\mathcal{G}}_{s+1}\\
&\times\hat{x}_{s+1|s}\hat{x}_{s+1|s}^T\bar{\mathcal{G}}_{s+1}^T)\mathcal{K}_{s+1}^T + (1+\beta_5^{-1})\\
&\times\mathcal{K}_{s+1}(\Lambda_{s+1}\circ\bar{\mathcal{D}}_{s+1}V_{s+1}\bar{\mathcal{D}}_{s+1}^T)\mathcal{K}_{s+1}^T. \qquad (21)
\end{aligned}
$$

Along the similar lines in [11], we can obtain that $\Gamma_{s+1|s+1} \le \bar{\aleph}_{s+1|s+1}$ and the proof is complete. ■

By now, we have obtained the UB $\bar{\aleph}_{s+1|s+1}$ for the EEC $\Gamma_{s+1|s+1}$. Next, we intend to compute the desired time-varying estimator gain through minimizing the obtained UB $\bar{\aleph}_{s+1|s+1}$ at each time s.

Theorem 2: The trace of the UB of the EEC $\bar{\aleph}_{s+1|s+1}$ is minimized by the following estimator gain matrix:

$$
\mathcal{K}_{s+1} = (1+\beta_2+\beta_3)\bar{\aleph}_{s+1|s}\bar{\mathcal{G}}_{s+1}^T\Psi_{s+1}^{-1} \qquad (22)
$$

where

$$
\begin{aligned}
\Psi_{s+1} \triangleq\ & (1+\beta_2+\beta_3)\Lambda_{s+1}^2\bar{\mathcal{G}}_{s+1}\bar{\aleph}_{s+1|s}\bar{\mathcal{G}}_{s+1}^T + (1+\gamma)\\
&\times(1+\beta_3^{-1}+\beta_4^{-1})((\Lambda_{s+1}-\Lambda_{s+1}^2)\circ\bar{\mathcal{G}}_{s+1}\bar{\aleph}_{s+1|s}\bar{\mathcal{G}}_{s+1}^T)\\
&+(1+\beta_3^{-1}+\beta_4^{-1})(1+\gamma^{-1})((\Lambda_{s+1}-\Lambda_{s+1}^2)\circ\bar{\mathcal{G}}_{s+1}\\
&\times\hat{x}_{s+1|s}\hat{x}_{s+1|s}^T\bar{\mathcal{G}}_{s+1}^T) + (1+\beta_2^{-1}+\beta_4+\beta_5)\\
&\times(\Lambda_{s+1}\circ(n_y\ell^2/4)I) + (1+\beta_5^{-1})\\
&\times(\Lambda_{s+1}\circ\bar{\mathcal{D}}_{s+1}V_{s+1|s}\bar{\mathcal{D}}_{s+1}^T). \qquad (23)
\end{aligned}
$$

Proof: The proof is straightforward based on Theorem 1 and is therefore omitted here for space saving. ■

An illustrative example: We intend to provide a simulation example to verify the availability of our developed fault estimation strategy in this section.

Consider a nonlinear TVS of the form (1) with the following parameters:

$$
\mathcal{A}_s = \begin{bmatrix} 0.380 & 0.01\cos(0.5s) & 0.022\\ 0.012 & 0.490 & 0.012\\ 0.017 & 0.025 & 0.400 \end{bmatrix},\ \mathcal{B}_s = \begin{bmatrix} 0.02\\ 0.01\\ 0.01 \end{bmatrix}
$$

$$
\mathcal{F}_s = \begin{bmatrix} 0.35\\ -0.23\\ 0.21 \end{bmatrix},\ l(x_s) = \begin{bmatrix} 0.17\sin(x_{1,s})\\ 0.16\sin(x_{2,s})\\ 0.17\sin(x_{3,s}) \end{bmatrix}
$$

$$
\mathcal{G}_s = [2 \quad 2+0.1e^{-0.5s}\cos(0.5s) \quad 2],\ \mathcal{D}_s = 0.01.
$$

The covariances of the process noise ω_s and the measurement noise ν_s are selected as $W_s = 0.25I$ and $V_s = 0.09I$, respectively. The nonlinear function $l(x_s)$ satisfies the constraint in Assumption 1 with $\Im = 0.17$. Suppose that the additive fault is $f_s = -1+0.01s$. The maximum limit for the energy storage is 3. Moreover, the probability distribution of h_s^1 is given by $p_0 = 0.200$, $p_1 = 0.346$ and $p_2 = 0.454$.

Set the initial energy unit and the maximum number of unit energy as $z_0 = 1$ and $S = 3$, respectively. According to (10), the values of Λ_s and the probability distribution of the sensor energy level θ_s are obtained.

Let the initial value of $\Gamma_{0|0}$ be I. The desired estimator gain is obtained by solving Theorems 1 and 2, the simulation results are presented in Figs. 1−3. Fig. 1 shows the values of h_s and Υ_s, Fig. 2 shows the actual state x_s and their estimates. Fig. 3 plots the fault f_s and the corresponding estimate, which indicates that our developed fault estimation performance is satisfactory.

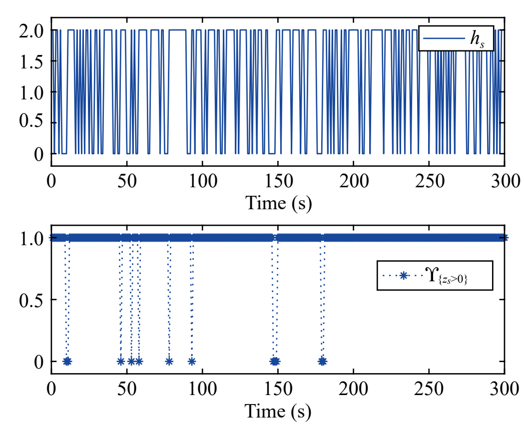

Fig. 1. The energy harvested and energy consumption.

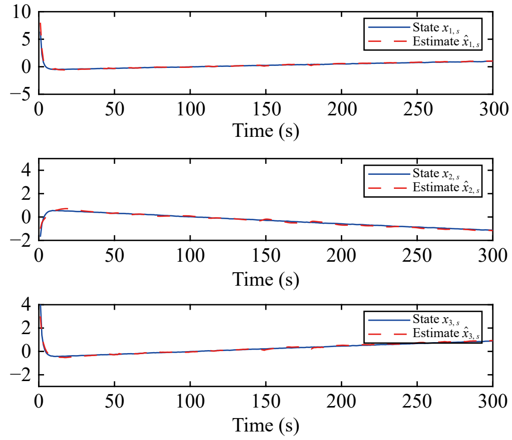

Fig. 2. The state x_s and its estimate $\hat{x}_s$.

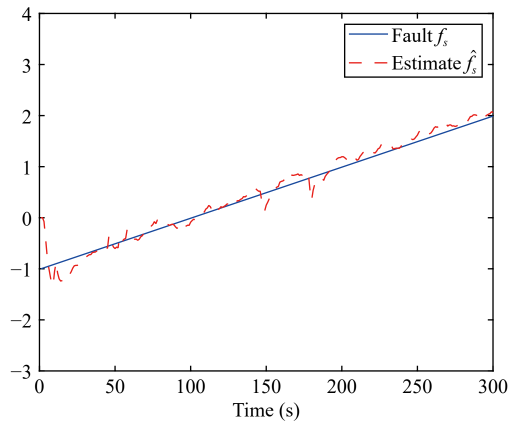

Fig. 3. The additive fault f_s and its estimate $\hat{f}_s$.

Conclusions: In this research, the problem of recursive fault estimation has been studied for nonlinear TVSs subject to EHS and signal UQE. According to the probability distribution of the energy harvest process, the probability of missing measurement has been calculated recursively. The fault signal under consideration has been modeled by a time-varying function whose second order difference is assumed to be zero. A recursive fault estimator has been developed to generate the state estimates and fault estimate at each time instant. Then, the UB of the resultant EEC has been derived in terms of two coupled differences equations. The required time-varying estimator gain has been computed recursively by minimizing the UB of the EEC. A numerical example has been addressed to examine the availability of the developed fault estimation method. Future research directions include the fault estimator design issue over sensor networks with EHS [12]–[15].

Acknowledgments: This work was supported in part by the National Natural Science Foundation of China (61873059, 61922024) and the Program of Shanghai Academic/Technology Research Leader (20XD1420100).

References

[1] E. Mousavinejad, X. Ge, Q.-L. Han, T. J. Lim, and L. Vlacic, "An ellipsoidal set-membership approach to distributed joint state and sensor fault estimation of autonomous ground vehicles," *IEEE/CAA J. Autom. Sinica*, vol. 8, no. 6, pp. 1107–1118, Jun. 2021.

[2] Y. Ju, D. Ding, X. He, and Q.-L. Han, "Consensus control of multi-agent systems using fault-estimation-in-the-loop: Dynamic event-triggered case", *IEEE/CAA J. Autom. Sinica*, DOI: 110.1109/JAS. 2021.1004386, 2021.

[3] Y. Liu, Z. Wang, and D. Zhou, "Resilient actuator fault estimation for discrete-time complex networks: A distributed approach", *IEEE Trans. Automatic Control*, DOI: 10.1109/TAC.2020.3033710, 2020.

[4] R. W. Brockett and D. Liberzon, "Quantized feedback stabilization of linear systems," *IEEE Trans. Automatic Control*, vol. 55, no. 7, pp. 1279–1289, Jul. 2000.

[5] M. Liu, D. W. C. Ho, and Y. Niu, "Robust filtering design for stochastic system with mode-dependent output quantization," *IEEE Trans. Signal Processing*, vol. 58, no. 11, pp. 6410–6416, Dec. 2010.

[6] S. Knorn and S. Dey, "Optimal energy allocation for linear control with packet loss under energy harvesting constraints," *Automatica*, vol. 77, pp. 259–267, Mar. 2017.

[7] J. Huang, D. Shi, and T. Chen, "Event-triggered state estimation with an energy harvesting sensor," *IEEE Trans. Automatic Control*, vol. 62, no. 9, pp. 4768–4775, Sep. 2017.

[8] Q. Li, Z. Wang, N. Li, and W. Sheng, "A dynamic event-triggered approach to recursive filtering for complex networks with switching topologies subject to random sensor failures," *IEEE Trans. Neural Networks and Learning Systems*, vol. 31, no. 10, pp. 4381–4388, Oct. 2020.

[9] Z. Gao, X. Liu, and M. Z. Q. Chen, "Unknown input observer-based robust fault estimation for systems corrupted by partially decoupled disturbances," *IEEE Trans. Industrial Electronics*, vol. 63, no. 4, pp. 2537–2547, Apr. 2016.

[10] B. Shen, Z. Wang, D. Wang, J. Luo, H. Pu, and Y. Peng, "Finite-horizon filtering for a class of nonlinear time-delayed systems with an energy harvesting sensor," *Automatica*, vol. 100, no. 2, pp. 144–152, Feb. 2019.

[11] J. Hu, Z. Wang, H. Gao, and L. K. Stergioulas, "Extended Kalman filtering with stochastic nonlinearities and multiple missing measurements," *Automatica*, vol. 48, no. 9, pp. 2007–2015, Sep. 2012.

[12] X. Ge, Q.-L. Han, M. Zhong, and X.-M. Zhang, "Distributed Krein space-based attack detection over sensor networks under deception attacks", *Automatica*, vol. 109, Article No. 108557, Nov. 2019.

[13] X. Ge, S. Xiao, Q.-L. Han, X.-M. Zhang, and D. Ding, "Dynamic event-triggered scheduling and platooning control co-design for automated vehicles over vehicular Ad-Hoc networks," *IEEE/CAA J. Autom. Sinica*, vol. 9, no. 1, pp. 31–46, Jan. 2022.

[14] X. Wang, D. Ding, X. Ge, and Q.-L. Han, "Neural-network-based control for discrete-time nonlinear systems with denial-of-service attack: The adaptive event-triggered case", *Int. Journal of Robust and Nonlinear Control*, DOI: 10.1002/rnc.5831, 2021.

[15] S. Liu, Z. Wang, G. Wei, and M. Li, "Distributed set-membership filtering for multi-rate systems under the Round-Robin scheduling over sensor networks," *IEEE Trans. Cybernetics*, vol. 50, no. 5, pp. 1910–1920, May 2020.