Letter

A Sandwich Control System with Dual Stochastic Impulses

Chuanlin Liao, Dan Tu, Yuming Feng, Wei Zhang, Zitao Wang, and B. O. Onasanya

Dear editor,

Along with the progress of science and technology and the development of social civilization, control system brings an increasingly significant function in daily life. The application field of control system is very wide, for instance, in mobile technology [1], artificial earth satellite [2], pest control [3], etc.

Ribeiro [4] first put forward the concept of random pulse in 1967. At present, impulsive control is used in networked control [5], secure communication [6], etc. In the 21st century, the impulsive control has been used in synchronization of coupled system, intelligent fault identification, image encryption, etc. [7]–[9]. Meanwhile, alternate control is often applied to traffic, irrigation of crop and current switching control [10]–[12], etc. Adaptive control is often used in robots [13], neural networks [14], etc.

At present, there are lots of means to stabilize a nonlinear system. For example, impulsive control [1]–[9], [15]–[17], alternate control [10]–[12], adaptive control [13], [14], [18], etc. In this letter, the goal is to devise some superior control systems after researching the general means of system control being used currently.

In some cases, the state is complicated, so the exact state of the system is not known. In this letter, in order to stabilize the current system, we propose a new method, which is called sandwich control system with dual stochastic impulses (SCSDSI). By derivation, it can be obtained that the exponential stability criterion of the system is in terms of a set of linear matrix inequalities (LMIs). Fig.1 reveals the operating principle of SCSDSI. In the first portion of the cycle, there exists a continuous input $C_1x(t)$, it ends stochastically, and an impulse H_1 is imposed after $C_1x(t)$. The second portion also has $C_2x(t)$ and H_2 in a similar way to the first one. In the third portion of the cycle, there also exists a continuous input $C_3x(t)$, but no impulse anymore. At the end of this letter, the use of this method controls the Chua's oscillator [19].

Problem statement: A classical nonlinear system is of the form

$$\begin{cases}\dot{x} = Gx(t) + f(x(t)) + \mu(t)\\ x(t_0) = x_0\end{cases} \tag{1}$$

where $x \in \mathbb{R}^n$ is the state vector, $f : \mathbb{R}^n \to \mathbb{R}^n$ is a continuous nonlinear function which satisfies $f(0) = 0$. Suppose that there is a diagonal matrix $L = \mathrm{diag}(\nu_1, \nu_2, \ldots, \nu_n) \geq 0$ so that $\|f(x)\|^2 \leq x^T Lx$ for any $x \in \mathbb{R}^n$. Concurrently, $Gx(t)$ is the linear part of the system, $f(x(t))$ is the nonlinear disturbance, and $\mu(t)$ is the outside input of system (1). The initial condition of system (1) is $x(t_0) = x_0$.

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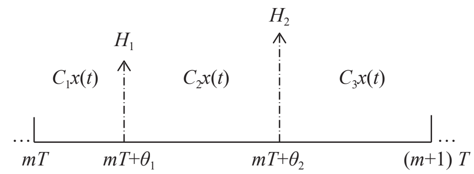


Fig. 1. Operating principle of SCSDSI within one control cycle T.

Let T be the control cycle of the system. m is a nonnegative integer. In order to stabilize the origin of system (1) by SCSDSI, T is divided into three parts:

- From mT to $mT+\theta_1$, $\mu(t) = C_1x(t)$ is installed, where $C_1 \in \mathbb{R}^{n\times n}$ is a constant matrix, and at time $mT+\theta_1$, a stochastic impulse H_1 occurs;
- From $mT+\theta_1$ to $mT+\theta_2$, $\mu(t) = C_2x(t)$ is installed, where $C_2 \in \mathbb{R}^{n\times n}$ is a constant matrix, and at time $mT+\theta_2$, a stochastic impulse H_2 occurs;
- From $mT+\theta_2$ to $(m+1)T$, $\mu(t) = C_3x(t)$ is installed, where $C_3 \in \mathbb{R}^{n\times n}$ is a constant matrix.

Through the above settings, system (1) can be described as

$$\begin{cases}\dot{x} = Gx(t) + f(x(t)) + C_1x(t), & mT < t < mT+\theta_1\\ x(t) = x(t^-) + H_1x(t^-), & t = mT+\theta_1\\ \dot{x} = Gx(t) + f(x(t)) + C_2x(t), & mT+\theta_1 < t < mT+\theta_2\\ x(t) = x(t^-) + H_2x(t^-), & t = mT+\theta_2\\ \dot{x} = Gx(t) + f(x(t)) + C_3x(t), & mT+\theta_2 < t \leq (m+1)T\end{cases} \tag{2}$$

where C_1, C_2, C_3, H_1, $H_2 \in \mathbb{R}^{n\times n}$ are constant matrices and $T > 0$ is the control cycle. Our purpose is to find conditions to make system (2) stable. So, it is of great necessity to determine C_1, C_2, C_3, H_1, H_2, θ_1, θ_2 and T.

In this letter, the maximum eigenvalue, the minimum eigenvalue and the transpose of a square matrix P_1 are denoted respectively by $\lambda_M(P_1)$, $\lambda_m(P_1)$ and P_1^T. The Euclidean norm of vector $x \in \mathbb{R}^n$ is denoted as $\|x\|$. When $P_1^T = P_1 > 0$ it is considered that the matrix P_1 is symmetric positive-definite while, if $P_1^T = P_1 < 0$, it is regarded that the matrix is symmetric negative-definite. Furthermore, $P_1^T = P_1 \geq 0$ is regarded that the matrix issymmetric positive semi-definite and $P_1^T = P_1 \leq 0$ is regarded that the matrix is symmetric negative semi-definite. $f(x(b^-))$ is defined as

$$f(x(b^-)) = \lim_{t\to b^-} f(x(t)).$$

Theorem 1: If there is a symmetric and positive definite matrix $P_1 \in \mathbb{R}^{n\times n}$ and real numbers $T > 0$, $g_1 > 0$, $g_2 > 0$, $g_3 > 0$, $\theta_1 > 0$, and $\theta_2 > \theta_1$ such that the following inequalities hold:

$$P_1G + G^TP_1 + P_1C_1 + C_1^TP_1 + \vartheta_1P_1^2 + \vartheta_1^{-1}L + g_1P_1 \leq 0$$

$$P_1G + G^TP_1 + P_1C_2 + C_2^TP_1 + \vartheta_2P_1^2 + \vartheta_2^{-1}L + g_2P_1 \leq 0$$

$$P_1G + G^TP_1 + P_1C_3 + C_3^TP_1 + \vartheta_3P_1^2 + \vartheta_3^{-1}L + g_3P_1 \leq 0$$

$$g_1\theta_1 + g_2(\theta_2-\theta_1) + g_3(T-\theta_2) - \ln\lambda_1 - \ln\lambda_2 > 0$$

then, system (2) has global exponential stability.

Proof: The Lyapunov function constructed is shown as follow:

$$V_1(x(t)) = x^T(t) P_1 x(t) \tag{3}$$

thus, it can be obtained that

Since $P_1G + G^TP_1 + P_1C_2 + C_2^TP_1 + \vartheta_2 P_1^2 + \vartheta_2^{-1}L + g_2P_1 \le 0$, the following is obtained

$$\lambda_m(P_1)\|x(t)\|^2 \le V(x(t)) \le \lambda_M(P_1)\|x(t)\|^2. \tag{4}$$

Suppose $mT < t < mT + \theta_1$, then by (2) and (4), the following is obtained

$$\begin{aligned}\dot{V}_1(x) &= 2x^TP_1\dot{x}\\ &= x^T[P_1G + G^TP_1 + P_1C_1 + C_1^TP_1]x + 2x^TP_1f(x)\\ &\le x^T[P_1G + G^TP_1 + P_1C_1 + C_1^TP_1]x\\ &\quad + \vartheta_1 x^TP^2x + \vartheta_1^{-1}x^TLx\\ &= -g_1V_1(x) + x^T[P_1G + G^TP_1 + P_1C_1 + C_1^TP_1\\ &\quad + \vartheta_1P_1^2 + \vartheta_1^{-1}L + g_1P_1]x.\end{aligned}$$

Since $P_1G + G^TP_1 + P_1C_1 + C_1^TP_1 + \vartheta_1P_1^2 + \vartheta_1^{-1}L + g_1P_1 \le 0$, the following formula can be derived

$$\dot{V}_1(x) \le -g_1V(x).$$

Thus,

$$V_1(x(t)) \le V_1(x(mT)^+)\exp(-g_1(t-mT)) \tag{5}$$

where $mT < t < mT + \theta_1$.

Suppose $t = mT + \theta_1$, then

$$\begin{aligned}V_1(x)|_{t=mT+\theta_1} &= (x(t^-) + H_1x(t^-))^TP_1(x(t^-) + H_1x(t^-))\\ &= x(t^-)^T(I+H_1)^TP_1(I+H_1)x(t^-)\\ &\le \lambda_1V_1(x(t^-))\end{aligned} \tag{6}$$

where $\lambda_1 = \lambda_M((I+H_1)^TP_1(I+H_1))/\lambda_m(P_1)$.

Suppose $mT + \theta_1 < t < mT + \theta_2$, then

$$\begin{aligned}\dot{V}_1(x) &\le vx^T[P_1G + G^TP_1 + P_1C_2 + C_2^TP_1]x\\ &\quad + \vartheta_2x^TP_1^2x + \vartheta_2^{-1}x^TLx\\ &= -g_2V_1(x) + x^T[P_1G + G^TP_1 + P_1C_2 + C_2^TP_1\\ &\quad + \vartheta_2P^2 + \vartheta_2^{-1}L + g_2P_1]x.\end{aligned}$$

$$\dot{V}_1(x) \le -g_2V_1(x).$$

Thus,

$$\begin{aligned}V_1(x(t)) &\le \lambda_1V_1(x(mT+\theta_1)^-)\\ &\quad \exp(-g_2(t-mT-\theta_1))\end{aligned} \tag{7}$$

where $mT + \theta_1 < t < mT + \theta_2$.

Suppose $t = mT + \theta_2$, then

$$V_1(x)|_{t=mT+\theta_2} \le \lambda_2V_1(x(t^-)) \tag{8}$$

where $\lambda_2 = \lambda_M((I+H_2)^TP_1(I+H_2))/\lambda_m(P_1)$.

Suppose $mT + \theta_2 < t \le (m+1)T$, then,

$$\begin{aligned}\dot{V}_1(x) &\le x^T[P_1G + G^TP_1 + P_1C_3 + C_3^TP_1]x\\ &\quad + \vartheta_3x^TP_1^2x + \vartheta_3^{-1}x^TLx\\ &= -g_3V_1(x) + x^T[P_1G + G^TP_1 + P_1C_2 + C_3^TP_1\\ &\quad + \vartheta_3P_1^2 + \vartheta_3^{-1}L + g_3P_1]x.\end{aligned}$$

Since $P_1G + G^TP_1 + P_1C_3 + C_3^TP_1 + \vartheta_3P_1^2 + \vartheta_3^{-1}L + g_3P_1 \le 0$, the following is obtained

$$\dot{V}_1(x) \le -g_3V_1(x).$$

Thus,

$$\begin{aligned}V_1(x(t)) &\le \lambda_2V_1(x(mT+\theta_2)^-)\\ &\quad \exp(-g_3(t-mT-\theta_2))\end{aligned} \tag{9}$$

where $mT + \theta_1 < t \le (m+1)T$.

Similar to the method used in [17], we get.

Case 1: $m = 0$

Subcase 1: Suppose $0 < t < \theta_1$, then

$$V_1(x(t)) \le V_1((x(0))\exp(-g_1t)$$

hence,

$$V_1(x(\theta_1)^-) \le V_1((x(0))\exp(-g_1\theta_1).$$

Subcase 2: Suppose $t = \theta_1$, it can be obtained that

$$V_1(x(\theta_1)) \le \lambda_1V_1((x(0))\exp(-g_1\theta_1).$$

Subcase 3: Suppose $\theta_1 < t < \theta_2$, then

$$V_1(x(t)) \le \lambda_1V_1(x(0))\exp(-g_1\theta_1 - g_2(t-\theta_1))$$

and

$$V_1(x(\theta_2)^-) \le \lambda_1V_1(x(0))\exp(-g_1\theta_1 - g_2(\theta_2-\theta_1)).$$

Subcase 4: Suppose $t = \theta_2$, it can be obtained that

$$V_1(x(\theta_2)) \le \lambda_1\lambda_2V_1((x(0))\exp(-g\theta_1 - g(\theta_2-\theta_1)).$$

Subcase 5: Suppose $\theta_2 < t \le T$, then

$$\begin{aligned}V_1(x(t) &\le \lambda_2V_1((x(\theta_2)^-))\exp(-g_3(t-\theta_2))\\ &\le \lambda_1\lambda_2V(x(0))\exp(-g_1\theta_1\\ &\quad - g_2(\theta_2-\theta_1) - g_3(t-\theta_2))\end{aligned}$$

and

$$\begin{aligned}V_1(x(T)) &\le \lambda_1\lambda_2V_1(x(0))\exp(-g_1\theta_1\\ &\quad - g_2(\theta_2-\theta_1) - g_3(T-\theta_2)).\end{aligned}$$

Case $n+1 : m = n$.

Subcase 1: Suppose $nT < t < nT + \theta_1$, then

$$\begin{aligned}V_1(x(t)) &\le \lambda_1^n\lambda_2^nV_1(x(0))\exp(-ng_1\theta_1 - ng_2(\theta_2\\ &\quad - \theta_1) - ng_3(T-\theta_2) - g_1(t-nT)).\end{aligned} \tag{10}$$

Subcase 2: Suppose $t = nT + \theta_1$, then

$$\begin{aligned}V_1(x(t)) &\le \lambda_1^{n+1}\lambda_2^nV_1(x(0))\exp(-(n+1)g_1\theta_1\\ &\quad - ng_2(\theta_2-\theta_1) - ng_3(T-\theta_2)).\end{aligned} \tag{11}$$

Subcase 3: Suppose $nT + \theta_1 < t \le nT + \theta_2$, then

$$\begin{aligned}V_1(x(t)) &\le \lambda_1^{n+1}\lambda_2^nV_1(x(0))\exp(-(n+1)g_1\theta_1\\ &\quad - ng_2(\theta_2-\theta_1) - ng_3(T-\theta_2)\\ &\quad - g_2(t-nT-\theta_1)).\end{aligned} \tag{12}$$

Subcase 4: Suppose $t = nT + \theta_1$, then

$$\begin{aligned}V_1(x(t)) &\le \lambda_1^{n+1}\lambda_2^{n+1}V_1(x(0))\exp(-(n+1)g_1\theta_1\\ &\quad - (n+1)g_2(\theta_2-\theta_1) - ng_3(T-\theta_2)).\end{aligned} \tag{13}$$

Subcase 5: Suppose $nT + \theta_2 < t < (n+1)T$, then

$$\begin{aligned}V_1(x(t)) &\le \lambda_1^{n+1}\lambda_2^{n+1}V_1(x(0))\exp(-(n+1)g_1\theta_1\\ &\quad - (n+1)g_2(\theta_2-\theta_1) - ng_3(T-\theta_2)\\ &\quad - g_3(t-nT-\theta_2)).\end{aligned} \tag{14}$$

From (10) it can be obtained that if $nT < t \le nT + \theta_1$, and $t = nT + \theta_1$, then

$$\begin{aligned}V_1(x(t)) &\le \lambda_1^{n+1}\lambda_2^nV_1(x(0))\exp(-(n+1)g_1\theta_1\\ &\quad - ng_2(\theta_2-\theta_1) - ng_3(T-\theta_2))\\ &\le V_1(x(0))\exp(-(g_1\theta_1 + g_2(\theta_2-\theta_1)\\ &\quad + g_3(T-\theta_2) - \ln\lambda_1 - \ln\lambda_2)n\\ &\quad + \ln\lambda_1 - g_1\theta_1)).\end{aligned}$$

From (14) it is obtained that if $nT + \theta_1 < t \le (n+1)T$, and $t = (n+1)T$, then

$$\begin{aligned}V_1(x(t)) &\le \lambda_1^{n+1}\lambda_2^{n+1}V_1(x(0))\exp(-(n+1)g_1\theta_1\\ &\quad - (n+1)g_2(\theta_2-\theta_1) - ng_3(T-\theta_2)\\ &\quad - g_3(t-nT-\theta_2))\\ &\le V_1(x(0))\exp(-(g_1\theta_1 + g_2(\theta_2-\theta_1)\\ &\quad + g_3(T-\theta_2) - \ln\lambda_1 - \ln\lambda_2)(n+1)).\end{aligned}$$

By the above inequalities, and the conditions of Theorem 1, it is obtained that $V_1(x(t)) \to 0$ as $t \to \infty$. Therefore, $\lim_{t\to\infty} V_1(x(t)) = 0$. ■

Numerical results: The Chua's oscillator [19] is described as follow:

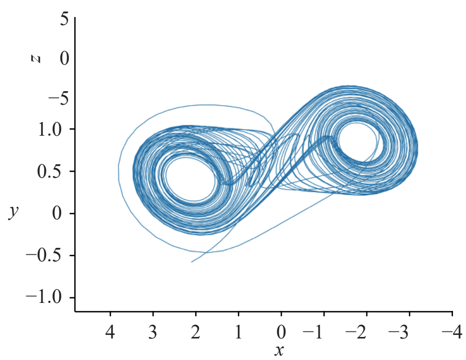
$$\begin{cases}\dot{x}_1 = \varpi(x_2 - x_2 - g_1(x_1))\\ \dot{x}_2 = x_1 - x_2 + x_3\\ \dot{x}_3 = \upsilon x_2\end{cases} \tag{15}$$

where ϖ and υ are two parameters, $g_1(x)$ is a piecewise linear characteristic function of Chua's diode. It can be defined as

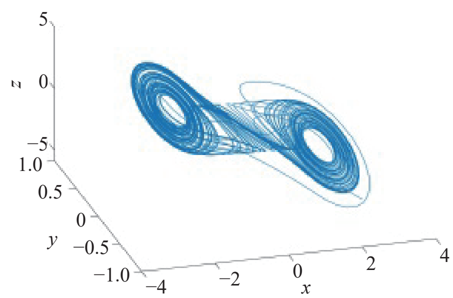
$$g_1(x) = ax + 0.5(b-a)(|x+1| - |x-1|). \tag{16}$$

Furthermore, a and b are two constants and $b < a \le 0$.

Then, we select the parameters $\varpi = 9.1156$, $\upsilon = 15.9946$, $b = -1.249\,05$, and $a = -0.757\,35$ which makes the Chua's oscillator chaotic at $x(0) = (2,-1,-2)^T$, as shown in Fig.2.



(a) Chaos of Chua's oscillator in two-dimensional plane



(b) Chaos of Chua's oscillator in three-dimensional space

Fig. 2. The chaotic phenomena of Chua's oscillator with $x(0) = (2,-1,-2)^T$.

Setting $L = \text{diag}(\varpi^2(b-a)^2, 0, 0)$, and selecting

$$C_1 = \text{diag}(-7,-8,-5)$$
$$C_2 = \text{diag}(-8,-7,-5)$$
$$C_3 = \text{diag}(-9,-8,-5)$$
$$H_1 = \begin{bmatrix} 0.08 & 0 & 0\\ 0 & -0.25 & 0\\ 0 & 0 & -0.4\end{bmatrix}$$
$$H_2 = \begin{bmatrix} 0.31 & 0 & -0.2\\ 0 & -0.5 & 0\\ 0 & 0 & -0.92\end{bmatrix}$$

with $T = 1$, solving the LMIs listed in Theorem 1, we obtain $\vartheta_1 = 7$, $\vartheta_2 = 8$, $\vartheta_3 = 9$, $\theta_1 = 0.0357$, $\theta_2 = 0.2321$, $g_1 = 4$, $g_2 = 5$, $g_3 = 6$, and

$$P_1 = \begin{bmatrix} 0.6120 & -0.1267 & 0.1338\\ -0.1267 & 0.9317 & -0.0032\\ 0.1338 & -0.0032 & 0.0793\end{bmatrix}.$$

Therefore, the Chua's system (15) is exponentially stable by Theorem 1. The time response curves of Chua's oscillator, using the proposed method, are shown in Fig. 3.

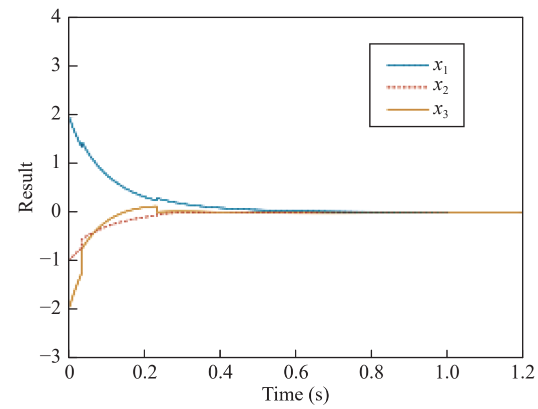


Fig. 3. The time response curves of Chua's oscillator by using SCSDSI.

Conclusions: In this letter, a new model of Sandwich control system with dual stochastic impulses (SCSDSI), where the exact time of occurrence of impulses is uncertain, was proposed. It established that the chaotic Chua's circuit can be controlled by this new method. In fact, by this method, a host of other nonlinear systems in robotics, electronics and other fields can be controlled.

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References

[1] H. H. Wilmer and J. M. Chein, "Mobile technology habits: Patterns of association among device usage, intertemporal preference, impulse control, and reward sensitivity," *Psychonomic Bulletin and Review*, vol. 23, pp. 1607–1614, 2016.

[2] V. V. Chazov, F. I. Ivandikov, A. A. Kuchumov, L. V. Kurakhtenkov, and V. R. Magsumov, "An algorithm for maintaining a given structure of an earth satellite constellation," in *Proc. Systems of Signal Synchronization, Generating and Processing in Telecommunications (SYNCHROINFO)*, 2019, pp. 1–4.

[3] X. Liang, Y. Pei, M. Zhu, and Y. Lv, "Multiple kinds of optimal impulse control strategies on plant pest predator model with eco-epidemiology," *Applied Mathematics and Computation*, vol. 287–288, pp. 1–11, 2016.

[4] S. T. Ribeiro, "Random-pulse machines," *IEEE Trans. Electronic Computers*, vol. EC-16, no. 3, pp. 261–276, 1967.

[5] Z. Yang, T. Huang, and L. Zhang, "On networked control of impulsive hybrid systems," *Computers and Mathematics with Applications*, vol. 61, no. 8, pp. 2076–2080, 2012.

[6] X. Yang, Z. Yang, and X. Nie, "Exponential synchronization of discontinuous chaotic systems via delayed impulsive control and its application to secure communication," *Communications in Nonlinear Science and Numerical Simulation*, vol. 19, no. 5, pp. 1529–1543, 2014.

[7] W. Zhang and J. Qi, "Synchronization of coupled memristive inertial delayed neural networks with impulse and intermittent control," *Neural Computing and Applications*, vol. 33, pp. 7953–7964, 2021.

[8] X. Zhao, Y. Qin, C. He, and L. Jia, "Intelligent fault identification for rolling element bearings in impulsive noise environments based on cyclic correntropy spectra and LSSVM," *IEEE Access*, vol. 8, pp. 40925–40938, 2020.

[9] H. Li, C. Li, D. Ouyang, and S. K. Nguang, "Impulsive synchronization of unbounded delayed inertial neural networks with actuator saturation and sampled-data control and its application to image encryption," *IEEE Trans. Neural Networks and Learning Systems*, vol. 32, no. 4,

pp. 1460–1473, 2020.

[10] J. Zhang, N. Naslavsky, and S. Caplan, “Rabs and EHDs: Alternate modes for traffic control,” *Bioscience Reports*, vol. 32, no. 1, pp. 17–23, 2012.

[11] S. Kang and J. Zhang, “Controlled alternate partial root-zone irrigation: Its physiological consequences and impact on water use efficiency,” *Journal of Experimental Botany*, vol. 55, no. 407, pp. 2437–2446, 2004.

[12] S. Liu, M. Saeedifard, and X. Wang, “Zero-current switching control of the alternate arm HVDC converter station with an extended overlap period,” *IEEE Trans. Industrial Electronics*, vol. 66, no. 3, pp. 2355–2365, 2019.

[13] D. Zhang, H. Yuan, and Z. Cao, “Environmental adaptive control of a snake-like robot with variable stiffness actuators,” *IEEE/CAA Journal of Automatica Sinica*, vol. 7, no. 3, pp. 745–751, 2020.

[14] Y. Ouyang, L. Dong, L. Xue, and C. Sun, “Adaptive control based on neural networks for an uncertain 2-DOF helicopter system with input deadzone and output constraints,” *IEEE/CAA Journal of Automatica Sinica*, vol. 6, no. 3, pp. 807–815, 2019.

[15] X. Zhang, H. Ma, W. Zuo, and M. Luo, “Adaptive control of discrete-time nonlinear systems using ITF-ORVFL,” *IEEE/CAA Journal of Automatica Sinica*, vol. 9, no. 3, pp. 556–563, 2022.

[16] Y. Feng, J. Yu, C. Li, T. Huang, and H. Che, “Linear impulsive control system with impulse time windows,” *Journal of Vibration and Control*, vol. 23, no. 1, pp. 111–118, 2017.

[17] Y. Feng, C. Li, and T. Huang, “Sandwich control systems with impulse time windows,” *Int. Journal of Machine Learning and Cybernetics*, vol. 8, pp. 2009–2015, 2017.

[18] X. Li, D. Peng, and J. Cao, “Lyapunov stability for Impulsive systems via event-triggered impulsive control,” *IEEE Trans. Automatic Control*, vol. 65, no. 11, pp. 4908–4913, 2020.

[19] L. P. Shil’nikov, “Chua’s circuit: rigorous results and future problems,” *Int. Journal of Bifurcation and Chaos*, vol. 4, no. 3, pp. 489–519, 1994.