The Effect of Tossing Coins in Omega-Automata

Christel Baier[1], Nathalie Bertrand[2], and Marcus Größer[1]

[1] Technische Universität Dresden, Faculty of Computer Science, Germany
[2] INRIA Rennes Bretagne Atlantique, France

Abstract. In this paper we provide a summary of the fundamental properties of probabilistic automata over infinite words. Such probabilistic automata are a variant of standard automata with Büchi or other ω-regular acceptance conditions, such as Rabin, Streett, parity or Müller, where the nondeterministic choices are resolved probabilistically. Acceptance of an infinite input word can be defined in different ways: by requiring that (i) almost all runs are accepting, or (ii) the probability for the accepting runs is positive, or (iii) the probability measure of the accepting runs is beyond a certain threshold. Surprisingly, even the qualitative criteria (i) and (ii) yield a different picture concerning expressiveness, efficiency, and decision problems compared to the nondeterministic case.

Introduction

Automata over infinite objects play a central role for verification purposes, reasoning about infinite games and logics that specify nondeterministic behaviors. Many variants of ω-automata have been studied in the literature that can be classified according to their inputs (e.g., words or trees), their acceptance conditions (e.g., Büchi, Rabin, Streett, Muller or parity acceptance) and their branching structure (e.g., deterministic, nondeterministic, or alternating). See, e.g., [9,18] for an overview of automata over infinite objects.

Although probabilistic finite automata (PFA) have attracted many researchers, see e.g. [8,12,13,15], probabilistic language acceptors for infinite words just have recently been studied. The formal definition of probabilistic ω-automata is the same as for nondeterministic ω-automata, the only difference being that all choices are resolved by probabilistic distributions. Acceptance of an infinite word $\sigma = a_1 a_2 a_3 \ldots$ can then be defined by imposing a condition on the probability of the accepting runs for σ. We consider here three types of accepted languages. The *probable semantics* requires positive probability for the accepting runs, the *almost-sure semantics* requires that the accepting runs for σ have probability 1, while the *threshold semantics* relies on some fixed threshold $\lambda \in]0,1[$ and requires that the acceptance probability is greater than λ.

Given the well-known fact that PFA are more expressive that NFA and that many relevant decision problems for PFA are undecidable, it is no surprise that PBA with the threshold semantics are more powerful than NBA and that the emptiness problem and other decision problems are undecidable for them. The definition of the accepted language under the probable semantics via the criterion *"the probability for the accepting runs is* > 0*"* appears to be the natural adaption of the definition of the accepted language of a nondeterministic automaton which relies on the criterion *"there is at least*

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one accepting run". One therefore might expect that probabilistic and nondeterministic ω-automata are rather close and enjoy similar properties. This, however, is not the case. The class of languages that are accepted by a PBA with the probable semantics strictly subsumes the class of ω-regular languages and it is closed under union, intersection and complementation. Furthermore, there are ω-regular languages that are recognizable by probable PBA of linear size while the sizes of smallest NBA for these languages grow exponentially. The price we have to pay for this extra power of probabilistic automata is that basic problems like checking emptiness, universality or equivalence are undecidable for PBA under the probable semantics.

The almost-sure semantics of PBA is "weaker" in the sense that each almost-sure PBA can be transformed into an equivalent PBA with the probable semantics, but not vice versa. Moreover, the class of languages that are recognizable by PBA with the almost-sure semantics does not cover the full class of ω-regular languages, it is not closed under complementation and contains non-ω-regular languages. On the positive side, the emptiness and universality problem for almost-sure PBA are decidable.

Organization. Section 1 recalls the definition of nondeterministic ω-automata with Büchi, Rabin or Streett acceptance conditions and introduces their probabilistic variants. Results on the expressiveness and efficiency of probabilistic Büchi automata are summarized in Section 2. Composition operators for PBA and probabilistic automata with Rabin and Streett acceptance are considered in Section 3. Decision problems for PBA will be discussed in Section 4. Finally, Section 5 contains some concluding remarks.

The material of this paper is a summary of the results presented in the papers [2,3]. Further details can be found there and in the thesis by Marcus Größer [10].

1 Probabilistic ω-Automata

We assume some familiarity with classical nondeterministic automata over finite or infinite words; see e.g. [9,18]. We just recall some basic concepts of nondeterministic ω-automata, and then present the definition of probabilistic ω-automata.

Definition 1 (Nondeterministic ω-automata). *A nondeterministic ω-automaton is a tuple* $\mathcal{N} = (Q, \Sigma, \delta, Q_0, \mathsf{Acc})$, *where*

- Q *is a finite nonempty set of states,*
- Σ *is a finite nonempty input alphabet,*
- $\delta : Q \times \Sigma \to 2^Q$ *is a transition function that assigns to each state q and letter* $a \in \Sigma$ *a (possibly empty) set* $\delta(q,a)$ *of states,*
- $Q_0 \subseteq Q$ *is the set of* initial states*,*
- Acc *is an* acceptance condition *(which will be explained later).*

$\mathcal{N}$ *is called deterministic if* $|Q_0| = 1$ *and* $|\delta(q,a)| = 1$ *for all* $q \in Q$ *and* $a \in \Sigma$.

Given an input word $\sigma = a_1 a_2 a_3 \ldots \in \Sigma^\omega$, a run for σ in $\mathcal{N}$ is a maximal state-sequence $\pi = q_0 q_1 q_2 \ldots$ such that $q_0 \in Q_0$, $q_{i+1} \in \delta(q_i, a_{i+1})$ for all $i \geq 0$. Maximality means that either π is infinite or ends in state q_n if $\delta(q_n, a_{n+1}) = \emptyset$. Each finite run $q_0 q_1 \ldots q_n$