A Moderately Exponential Time Algorithm for Full Degree Spanning Tree

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Abstract. We consider the well studied Full Degree Spanning Tree problem, a NP-complete variant of the Spanning Tree problem, in the realm of moderately exponential time exact algorithms. In this problem, given a graph G, the objective is to find a spanning tree T of G which maximizes the number of vertices that have the same degree in T as in G. This problem is motivated by its application in fluid networks and is basically a graph-theoretic abstraction of the problem of placing flow meters in fluid networks. We give an exact algorithm for Full Degree Spanning Tree running in time $\mathcal{O}(1.9172^n)$. This adds Full Degree Spanning Tree to a very small list of "non-local problems", like Feedback Vertex Set and Connected Dominating Set, for which non-trivial (non brute force enumeration) exact algorithms are known.

1 Introduction

The problem of finding a spanning tree of a connected graph arises at various places in practice and theory, like the analysis of communication or distribution networks, or modeling problems, and can be solved efficiently in polynomial time. On the other hand, if we want to find a spanning tree with some additional properties like maximizing the number of leaves or minimizing the maximum degree of the tree, the problem becomes NP-complete. This paper deals with one of the NP hard variants of Spanning Tree, namely Full Degree Spanning Tree from the view point of moderately exponential time algorithms.

> Full Degree Spanning Tree (FDST): Given an undirected connected graph $G = (V, E)$, find a spanning tree T of G which maximizes the number of vertices of *full degree*, that is the vertices having the same degree in T as in G.

The FDST problem is motivated by its applications in water distribution and electrical networks [15,16,17,18]. Pothof and Schut [18] studied this problem in the context of water distribution networks where the goal is to determine or control the flows in the network by installing and using a small number of flow meters. It turns out that to measure flows in all pipes, it is sufficient to find a full degree spanning tree T of the network and install flow meters (or pressure

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gauges) at each vertex of T that does not have full degree. We refer to [1,4,11] for a more detailed description of various applications of FDST.

The FDST problem has attracted a lot of attention recently and has been studied extensively from different algorithmic paradigms, developed for coping with NP-completeness. Pothof and Schut [18] studied this problem first and gave a simple heuristic based algorithm. Bhatia et al. [1] studied it from the view point of approximation algorithms and gave an algorithm of factor $\mathcal{O}(\sqrt{n})$. On the negative side, they show that FDST is hard to approximate within a factor of $\mathcal{O}(n^{\frac{1}{2}-\epsilon})$, for any $\epsilon > 0$, unless $coR = NP$, a well known complexity-theoretic hypothesis. Guo et al. [10] studied the problem in the realm of parameterized complexity and observed that the problem is W[1]-complete. The problem which is dual to FDST is also studied in the literature, that is the problem of finding a spanning tree that minimizes the number of vertices not having full degree. For this dual version of the problem, Khuller et al [11] gave an approximation algorithm of factor $2 + \epsilon$ for any fixed $\epsilon > 0$, and Guo et al. [10] gave a fixed parameter tractable algorithm running in time $4^k n^{\mathcal{O}(1)}$. FDST has also been studied on special graph classes like planar graphs, bounded degree graphs and graphs of bounded treewidth [4]. The goal of this paper is to study Full Degree Spanning Tree in the context of moderately exponential time algorithms, another coping strategy to deal with NP-completeness. We give a $\mathcal{O}(1.9172^n)$ time algorithm breaking the trivial $2^n n^{\mathcal{O}(1)}$ barrier.

Exact exponential time algorithms have an old history [5,14] but the last few years have seen a renewed interest in the field. This has led to the advancement of the state of the art on exact algorithms and many new techniques based on Inclusion-Exclusion, Measure & Conquer and various other combinatorial tools have been developed to design and analyze exact algorithms [2,3,7,8,12]. Branch & Reduce has always been one of the most important tools in the area but its applicability was mostly limited to 'local problems' (where the decision on one element of the input has direct consequences for its neighboring elements) like Maximum Independent Set, SAT and various other problems, until recently. In 2006, Fomin et al.[9] devised an algorithm for Connected Dominating Set (or Maximum Leaf Spanning Tree) and Razgon [19] for Feedback Vertex Set combining sophisticated branching and a clever use of measure. Our algorithm adheres to this machinery and adds an important real life problem to this small list. We also need to use an involved measure, which is a function of the number of vertices and the number of edges to be added to the spanning tree, to get the desired running time.

2 Preliminaries

Let G be a graph. We use $V(G)$ and $E(G)$ to denote the vertices and the edges of G respectively. We simply write V and E if the graph is clear from the context. For $V' \subseteq V$ we define an *induced subgraph* $G[V'] = (V', E')$, where $E' = \{uv \in E : u, v \in V'\}$.